

A criterion for admissible singularities in brane world

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Abstract

When gravity couples to scalar fields in Anti-de Sitter space, the geometry becomes non-AdS and develops singularities generally. We propose a criterion that the singularity is physically admissible if the integral of the on-shell Lagrangian density over the finite range is finite everywhere. For all classes of the singularities studied here, the criterion suggested in this paper coincides with an independent proposal made by Gubser that the potential should be *bounded from above* in the solution. This gives a reason why Gubser's conjecture works.

I. INTRODUCTION

The brane world scenario has been extensively studied for last several years. The discovery of D-brane [1] in string theory changed the conventional viewpoint that the spacetime on which gauge fields propagate and gravitons propagate should be the same. The gauge fields can be confined on D-brane; the fact that gauge fields and gravitons can propagate in different spacetimes allowed us to look at traditional problems in entirely new ways. The gauge hierarchy problem, which has been a central issue in particle physics for several decades, is now posed differently as why the Planck scale is so high (gravity is so weak) compared to the electroweak scale (gauge interactions) [2–4]. A large extra dimension [2] diluting gravity explains the gauge hierarchy if a natural mechanism to stabilize the radion at large values can be found. Alternatively, Randall and Sundrum [3] proposed that if the extra dimension is negatively curved (Anti-de Sitter) and if we live on the negative tension brane, we can explain the huge discrepancy between the Planck scale and the electroweak scale without invoking large extra dimension. As an extension of this work, they also showed that the gravity can be confined [4] even for the extra dimension of infinite size if the tension of the brane and the bulk cosmological constant has a special relation.

While all the above scenarios have been invented to explain the gauge hierarchy problem, the cosmological constant problem still remains a serious conundrum. If the fine tuning between the bulk cosmological constant and the brane tension is incomplete, we get effective 4 dimensional cosmological constant proportional to the mismatch [5–8]. In this paper we provide no further insights to this issue and simply assume the fine tuning allowing us to restrict our attention only to the space-time geometries with 4 dimensional Poincare invariance.

The generalization of Randall-Sundrum (RS) setup has been studied extensively. For example, see [9,10]. The simplest interesting extension of RS setup is to consider bulk scalar fields coupled to gravity. First of all, known string theories contain various scalar fields such as dilaton, moduli and axions. As a first step toward the stringy generalization of RS scenario, the inclusion of the bulk scalar fields appears unavoidable. Furthermore, in two brane scenario [3], bulk scalar field is necessary for the radion stabilization [11,12]; without stabilization mechanism, pure gravity reveals the instability of the two brane systems [7]. One also notes that the bulk cosmological constant can be naturally generalized to the potential of scalar fields. In this paper we consider a gravity coupled to a single bulk scalar field.

According to the AdS/CFT correspondence [13–15], supergravity in AdS_{D+1} is dual to D dimensional conformal field theory. Extensions of the AdS/CFT correspondence to the duality between supergravities in non- AdS backgrounds and the field theories off the conformal fixed points have been suggested in supersymmetric context [16–23]. One of the general properties of these models is the appearance of naked singularities in the IR region of the background geometry. A concrete explanation for the appearance of the singularity can be found in [24]. The explanation is based on the analogy with inflation. Most inflationary scenario has a scalar field, inflaton, whose potential has a minimum or degenerate minima with vanishing vacuum energy implying that the potential is globally nonnegative. Though there are two types of solutions (inflating and “deflating”) due to time reversal symmetry in the Einstein equation, we are interested in inflating solutions in which the affine connection

terms act as a frictional force. With the aid of the friction, the inflaton settles down to a minimum. The minimum of the potential is always an attractor of the system in inflating models and we recover asymptotically de Sitter geometry. Just as we consider the motion of inflaton along the time direction, we consider the change of the bulk scalar field along the y (extra dimension) direction. Starting from an initial position $y = 0$, the scalar field develops a y -dependent profile determined by the equations of motion. Since we are interested in the brane world scenario whose D dimensional Newton constant is finite, only the decreasing warp factor should be considered. The following discussions are mainly independent of the presence of the branes and can be applied to whole AdS geometry in the same way. Now the situation is analogous to that of the “deflating” solution or time-reversed inflation. The crucial difference is that the affine connection terms act as an *anti-friction*. They prevent the scalars from settling down at a stationary point of the potential except when the initial condition has been precisely chosen to do it. The generic final destiny of the scalar is to roll up or down to infinity, producing a singularity. If this happens at finite $y = y_c$, there is a naked singularity.

These singularities appear in generic situations. Here, we propose a criterion that determines which types of the singularities are physically acceptable.

If the integral of the on-shell Lagrangian density over the finite range of y , whose least upper bound is $y = y_c$, is finite, the singularity at $y = y_c$ is physically admissible.

In [25], a different version of the criterion on the physically acceptable singularities was given.

Large curvatures in scalar coupled gravity with four dimensional Poincare invariant solution are allowed only if the scalar potential is bounded above in the solution.

The main observation of this paper is that the above two criteria that apparently look independent are equivalent for large class of known interesting examples. We expect that the two criteria are equivalent in general cases, even if we do not have a rigorous mathematical proof yet. The conjecture in [25] was based on the detailed study of the known supergravity examples. For the model having the potential unbounded from above, it has been shown that we always encounter a pathological problem when we see the singularity in the dual field theory or when we want to resolve it by lifting it to higher dimensions in string theory.

There are other criteria on the singularities [26–28,21]. In [26,27], the unitarity condition at the singularity is used to probe which types of singularities are harmless. In [21], the g_{00} component of the metric is required to be bounded above (or not to increase as we approach the singularity) for physically allowed singularities. It is likely that all these criteria share the common features. However, the connections among them are not clear, and to show the detailed connection is not discussed in this paper.

The rationale for our criterion comes from two sources.

First, to have a sensible semi-classical expansion around a given classical solution, it is necessary that the integral of the on-shell Lagrangian density over any finite volume be finite [29]. String theory in curved spacetime is far from complete. Curved spacetime is treated as the background, and at best we can take the semi-classical expansion around the background

as far as the gravity sector is concerned. In the absence of full quantum description of gravity, the only way on which we can rely is to use the semi-classical expansion. This approach is based on the belief that the semi-classical expansion grasps most of important physics. In other words the belief is that the difference between the fully quantized theory and the semi-classical one is negligible and the semi-classical treatment is trustworthy. However, once semi-classical expansion is not available, we can not trust the classical solution since the fully quantized theory is expected to be dramatically different from the classical one. The information gained from the geometry containing harmful singularities, which do not allow the semi-classical expansion, is therefore not trustable, and we expect the quantum effects of gravity will spoil the picture completely. In that case we should abandon classical general relativity description.

Second, to satisfy the consistency condition with putting a finite tension brane at the singularity, the finiteness of the on-shell Lagrangian density from $y = 0$ to $y = y_c$ is also required. There are several consistency conditions in the brane world scenario which become important in the presence of the singularity [30–32]. For the metric which keeps D dimensional Poincare invariance in $D + 1$ dimension, the consistency requires that the D dimensional energy density (or effective D dimensional cosmological constant) after integrating out the extra dimension y should vanish. Already in self-tuning model [33,34] it has been shown that we need an additional contribution to the effective 4 dimensional cosmological constant from the singularity to cancel the brane tension. In self-tuning model, the bulk cosmological constant is assumed to be zero and there is no other contribution to the 4 dimensional cosmological constant except the brane tension. This inconsistency problem can be overcome by putting additional brane at the position of the singularity such that the tension of it can cancel the tension of the visible brane at $y = 0$. In general, if the 4 dimensional energy density obtained by integrating over y from $y = 0$ to $y = y_c$ is finite, the consistency condition can be satisfied by putting additional brane on the singularity, and it leaves open a possibility that the singularity can be resolved in some ways. However, if the energy density integrated over y including the singularity is infinite, we can not make a consistent theory without introducing an artificial brane with infinite tension located at the singularity. This implies that the physical system, which will be obtained only after resolving the singularity, is entirely different from the classical configuration.

The criterion proposed here has nothing to do with supersymmetry. While all the known examples which have concrete realizations are supergravity models, our criterion is more fundamental and is also applicable to non-supersymmetric cases.

II. BASIC SETUP

We start from $D+1$ dimensional geometry with D Poincare invariant spacetime. The metric is

$$ds^2 = a^2(y)\eta_{\mu\nu}dx^\mu dx^\nu + dy^2, \quad (1)$$

and we define “Hubble parameter” as

$$H = \frac{a'}{a} = -h, \quad (2)$$

where the prime denotes the derivative with respect to y . This new “Hubble parameter” is defined to use an analogy with the inflation models where the derivatives are taken with respect to t instead of y .

Since we are interested in the geometry with decreasing warp factor such that D dimensional Newton constant can remain finite, H is always negative definite, and h is positive definite. ($h \geq 0$) In discussing the properties of the singularities, other parts cut out by the brane which approach Anti de Sitter space asymptotically do not affect the conclusion [25]. Once we start from negative H and go to the direction of decreasing warp factor, the holographic c-theorem guarantees that H remain negative definite.¹

D+1 dimensional action of scalar coupled gravity is

$$S = \int d^{D+1}x \sqrt{-g} \left(\frac{1}{2} R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right) + \int d^D x \sqrt{-g^{(D)}} \mathcal{L}_i, \quad (3)$$

with the unit of setting the fundamental scale $M_{D+1} = 1$. Single real scalar field is considered in this paper, but the generalization to the system including many scalars does not affect the result given in the paper since the singular behavior can be effectively described by single real scalar field. Let us assume $\mathcal{L}_0 = -V_0$ from now on and the brane is located at $y = y_0 = 0$. The brane is introduced in order to see the finiteness of the on-shell Lagrangian density more clearly. D dimensional energy density is well defined in the presence of the brane cutting the boundary part of AdS (asymptotically AdS) which is irrelevant in the discussion of the physics about the singularities. Einstein equations are summarized as two equations for the metric in eq. (1)

$$\begin{aligned} \phi'' + DH\phi' - \frac{dV}{d\phi} &= 0, \\ H &= -\sqrt{\frac{2}{D(D-1)} \left(\frac{\phi'^2}{2} - V \right)}. \end{aligned} \quad (4)$$

It is helpful to introduce new notation $U = -V$ with $h = -H$ since the new quantities U and h are mainly positive quantities. The Einstein equations are

$$\begin{aligned} \phi'' - Dh\phi' + \frac{dU}{d\phi} &= 0, \\ h &= \sqrt{\frac{2}{D(D-1)} \left(\frac{\phi'^2}{2} + U \right)}. \end{aligned} \quad (5)$$

It is clear that these two equations are exactly the same as the inflaton equations with Hubble parameter $-h$ and potential U (or y to t). Since the Hubble parameter $-h$ is negative, we

¹The absolute value h increases monotonically unless we introduce an object that violates the null energy condition.

should think of it as "deflation" or time-reversal solution to the usual inflation motion [24]. The comparison with the inflation models are well summarized in the table.

The equation looks simpler if we set $h = \sqrt{\frac{2}{D(D-1)}} \sqrt{G}$. We now have

$$\begin{aligned} G &= \frac{\phi'^2}{2} + U, \\ G' &= (\phi'' + \frac{dU}{d\phi})\phi', \end{aligned} \tag{6}$$

and the "Friedmann equation" becomes

$$G' - \sqrt{\frac{2D}{(D-1)}} G \phi'^2 = 0, \tag{7}$$

$$\phi'^2 = 2(G - U). \tag{8}$$

We can rewrite the "Friedmann equation" by eliminating ϕ'

$$G' - 2\sqrt{\frac{2D}{(D-1)}} G (G - U) = 0. \tag{9}$$

Furthermore, for the classical solution of the Einstein equation,

$$R = \partial_\mu \phi \partial^\mu \phi + \frac{2(D+1)}{D-1} V + \frac{2D}{D-1} V_0 \delta(y), \tag{10}$$

and if we put it again into the original action, the action is expressed only in terms of V independently of ϕ' .

$$S = \int d^{D+1}x \sqrt{-g} \left(\frac{2}{D-1} V \right) + \int d^D x \sqrt{-g^{(D)}} \left(\frac{1}{D-1} V_0 \right). \tag{11}$$

A particular boundary condition on the brane at $y = 0$ should be considered to have a consistent D dimensional Poincare invariant solution. The boundary jump condition is

$$H|_{-\epsilon}^\epsilon = -\frac{1}{D-1} V_0(\phi(y=0)) \tag{12}$$

$$\phi'|_{-\epsilon}^\epsilon = \frac{\partial V_0(\phi(y=0))}{\partial \phi}. \tag{13}$$

For positive definite h , the boundary jump condition is

$$h|_{-\epsilon}^\epsilon = \frac{1}{D-1} V_0(\phi(y=0)). \tag{14}$$

In the following analysis, V_0 is not specified and is assumed to be chosen to satisfy the junction condition of eq. (13) and (14) once we determine ϕ' and h at $y = 0$ (equivalently G or \sqrt{G} at $y = 0$). The boundary jump condition is necessary to give D dimensional Poincare invariant solution. Otherwise, we can not have a solution for the metric of eq. (1) with D dimensional Poincare invariance and end up obtaining different solutions, namely, dS_{D-1} or AdS_{D-1} [5–8].

III. CASE STUDIES

A. $U = \text{constant}$

Suppose the potential $-U$ is constant and negative ($U = U_0$ is positive). From the eq. (9), we have

$$\frac{dG}{\sqrt{G}(G-U)} = \sqrt{\frac{8D}{(D-1)}} dy. \quad (15)$$

The solution to the above equation is

$$\sqrt{G} = \sqrt{U_0} \coth \left(\sqrt{\frac{2D}{D-1}} (y_c - y) \right). \quad (16)$$

We are interested in $G \geq U_0$ and thus $y \leq y_c$. In this case, \sqrt{G} goes to infinity as y approaches y_c . Now ϕ can be calculated from U_0 and G .

$$\phi'^2 = 2(G - U) = \frac{2U_0}{\sinh^2 \left(\sqrt{\frac{2D}{D-1}} (y_c - y) \right)}, \quad (17)$$

$$\phi' = \frac{\pm \sqrt{2U_0}}{|\sinh \left(\sqrt{\frac{2D}{D-1}} (y_c - y) \right)|}.$$

$$\phi = \phi_0 \pm \sqrt{2U_0} \log \tanh \left(\sqrt{\frac{2D}{D-1}} (y_c - y) \right). \quad (18)$$

The “ D dimensional energy density” is defined by putting the scalar curvature R calculated from equations of motion into the Lagrangian and integrating over y (11). The next thing we can do is to check whether the “ D dimensional energy density” is finite or not. The precise D dimensional energy density is

$$I_D = \frac{2}{D-1} I + \frac{1}{D-1} V_0$$

$$S = \int d^D x I_D.$$

where

$$I = \int_0^{y_c} dy \sqrt{-g} V. \quad (19)$$

Since we are only interested in discriminating whether the “ D dimensional energy density” diverges or not, the numerical coefficients, the sign, and the finite contribution from the brane tension are neglected in the following discussions. Whether the quantity $|I|$ converges or diverges is the only important question in this paper. In the first example, the “ D dim. energy density” $|I|$ is

$$|I| = \int_0^{y_c} dy e^{-\int_0^y d\bar{y} \sqrt{\frac{2D}{(D-1)}} G} U, \quad (20)$$

and is finite. Therefore, the singularity developed in the flat potential satisfies the criterion, and we conclude that this singularity is admissible. The singularity appearing in the self-tuning model [33,34] belongs to this case. It is lucky if the exact solution is available which is the case here. In most cases it is not possible to obtain an analytic solution which is valid in all ranges from $y = 0$ to $y = y_c$. Nonetheless, it does not prevent us from checking whether some classes of the singularities satisfy the criterion or not once we know the behavior of the solutions near the singularity. The finiteness or infinity of the D dimensional energy density is determined only by the behavior in the neighborhood of the singularity. Thus in the following examples, we classify the solution by the singular behavior of the bulk scalar field and/or the metric and reconstruct the leading term of the potential when we approach the singularity.

As $y \rightarrow y_c$, the singular behavior can be characterized by

$$\phi' = \frac{A}{(y_c - y)^\alpha} \quad (21)$$

where A and α are arbitrary numbers. The limiting behavior of ϕ is then

$$\phi = \frac{A}{\alpha - 1} \frac{1}{(y_c - y)^{\alpha-1}}. \quad (22)$$

From the eq. (9), we get

$$\frac{G'}{2G} = \sqrt{\frac{D}{2(D-1)}} \phi'^2. \quad (23)$$

Now we are ready to get the limiting behavior of G by putting eq. (21) into eq. (7) and integrating over y

$$\sqrt{G} \sim \int dy \frac{1}{(y_c - y)^{2\alpha}}. \quad (24)$$

All the sub-leading corrections are omitted in the above expressions.

B. $\alpha > 1$

If $\alpha > 1$, then ϕ , ϕ' and G goes to ∞ as y goes to y_c . Integrating the previous equation,

$$\sqrt{G} \sim \frac{1}{(y_c - y)^{2\alpha-1}} \rightarrow \infty.$$

The exponent of the warp factor also goes to $-\infty$ as y goes to y_c

$$\lim_{y \rightarrow \infty} -\int_0^y d\bar{y} \sqrt{G} \sim \lim_{y \rightarrow \infty} -\frac{1}{(y_c - y)^{2\alpha-2}} \rightarrow -\infty. \quad (25)$$

From the information on ϕ' and G , we can construct U

$$\begin{aligned} U_{\text{leading}} &= G - \frac{\phi'^2}{2} \\ &= c_1 \frac{1}{(y_c - y)^{4\alpha-2}} - c_2 \frac{1}{(y_c - y)^{2\alpha}}, \end{aligned} \quad (26)$$

where c_i with $i = 1, 2$ are positive constants. Since we know the limiting behavior of ϕ itself, we can reconstruct U as a function of ϕ as ϕ goes to ∞ .

$$U_{\text{leading}} = \bar{c}_1 \left(\frac{\phi}{A}\right)^{n+2} - \bar{c}_2 \left(\frac{\phi}{A}\right)^n, \quad (27)$$

where $n = \frac{2\alpha}{\alpha-1} > 2$ is positive and \bar{c}_i are also positive constants. Thus in the limit of $\phi \rightarrow \pm\infty$, the first term dominates and determine the asymptotic form of the potential as

$$U_{\text{leading}} = \bar{c}_1 \left(\frac{\phi}{A}\right)^{n+2}. \quad (28)$$

For even n , A^{-n} is positive and U is bounded from below as $\phi \rightarrow \pm\infty$. For odd n , we have to consider two cases. First, $A > 0$. U is bounded from below as $\phi \rightarrow \infty$. Second, $A < 0$. We start from some ϕ_0 and as $y \rightarrow y_c$, $\phi \rightarrow -\infty$ and U is also bounded from below. This observation is very crucial. For odd n , the entire shape of U for ϕ is not bounded from below. However, as long as the solution is concerned, the U is bounded from below. Therefore, when $\alpha > 1$, for both $A > 0$ and $A < 0$, the potential V is bounded from above in the solution. For $\alpha > 1$, in the limit of $y_c - y \rightarrow 0^+$, the first term dominates in eq. (26) and U is always bounded from below. Thus the potential V is always bounded from below for $\alpha > 1$.

The D dimensional energy density for $\alpha > 1$ is

$$\lim_{y \rightarrow y_c} \int_0^y d\bar{y} e^{-\int_0^{\bar{y}} d\tilde{y} \sqrt{\frac{2D}{(D-1)}} G} U, \quad (29)$$

and is finite since $\lim_{x \rightarrow \infty} e^{-x^{2\alpha-2}} x^{4\alpha-2}$ is finite where $x \sim \frac{1}{y_c - y}$.

We can conclude that for $\alpha > 1$ the potential V is bounded from above in the solution and D dimensional energy density is finite.

C. $\alpha = 1$

This marginal case is very interesting because the borderline of the criterion lies here. The limiting behavior of the scalar field near the singularity is

$$\begin{aligned} \phi' &= \frac{A}{y_c - y} \\ \phi &= -A \log(y_c - y), \end{aligned} \quad (30)$$

where the sub-leading terms are omitted. From the equation 9, we get

$$\begin{aligned}
\sqrt{G} &= \sqrt{\frac{D}{2(D-1)}} A^2 \int dy \frac{1}{(y_c - y)^2} \\
&= \sqrt{\frac{D}{2(D-1)}} A^2 \frac{1}{(y_c - y)} + \text{nonsingular part.}
\end{aligned} \tag{31}$$

The next step is to get U from G and ϕ' from eq. (8)

$$\begin{aligned}
U &= G - \frac{\phi'^2}{2} \\
&= \frac{1}{2} \left(\frac{D}{D-1} A^2 - 1 \right) \frac{A^2}{(y_c - y)^2}.
\end{aligned} \tag{32}$$

Since we know the limiting behavior of U and ϕ , we can reformulate the leading term of $U(\phi)$ as

$$U(\phi)_{\text{leading}} = \frac{1}{2} A^2 \left(\frac{D}{D-1} A^2 - 1 \right) e^{\frac{2}{A}\phi} = \frac{A^2}{2} \zeta e^{\frac{2}{A}\phi}, \tag{33}$$

where

$$\zeta = \frac{D}{D-1} A^2 - 1. \tag{34}$$

We can extract the condition for U bounded from below. For $\zeta > 0$ ($|A| > \sqrt{\frac{D-1}{D}}$), U is bounded from below (the potential V is bounded from above). For $\zeta < 0$ ($|A| < \sqrt{\frac{D-1}{D}}$), U is not bounded from below (the potential V is not bounded from above).

To evaluate D dimensional energy density, first we integrate \sqrt{G} . The factor appearing in the exponent is

$$-\int_0^y dy' \sqrt{\frac{2D}{D-1}} G = \frac{D}{D-1} A^2 \log(y_c - y) + \text{finite terms}, \tag{35}$$

and D dimensional energy density for $\zeta \neq 0$ is

$$\begin{aligned}
\lim_{y \rightarrow y_c} \int_0^y d\bar{y} (y_c - \bar{y})^{\frac{DA^2}{D-1}} U &= \lim_{y \rightarrow y_c} \int_0^y d\bar{y} \frac{A^2}{2} \zeta (y_c - \bar{y})^{\zeta-1} \\
&= \lim_{y \rightarrow y_c} -\frac{A^2}{2} (y_c - y)^\zeta + \text{finite terms}.
\end{aligned} \tag{36}$$

For $\zeta > 0$, D dimensional energy density is finite. For $\zeta < 0$, D dimensional energy density diverges. We can further confirm the relation between two conditions.

It should be stressed that our criterion is different from the criterion to avoid the timelike naked singularity. The metric can be expressed explicitly near the singularity

$$\left(\frac{1}{y_c - y} \right)^{\frac{2A^2}{D-1}} (-dt^2 + d\vec{x}^2) + dy^2 \tag{37}$$

which clearly shows that for $|A| \geq \sqrt{D-1}$, the naked singularity is null singularity and it takes infinite time t to arrive at the singularity y_c . For $\sqrt{D-1} > |A| > \sqrt{\frac{D-1}{D}}^2$, even if the solution satisfies the criterion, we have a naked timelike singularity. Though Cauchy problem appears ill-defined in the presence of the timelike singularity, we cannot rule out this case. Coulomb branch solution of $D = 4$ in [25] is a definite example which has a timelike naked singularity but can be resolved without facing the pathological problems (except the unphysical Coulomb branch which can be ruled out also by our criterion). The criterion given in this paper is a more refined constraint and even some types of timelike naked singularities are admissible according to our criterion. We have to mention that a clear physical explanation of why we can have a sensible physical theory even if Cauchy problem is ill-posed is missing now.

D. $\alpha < 1$

It is easy to show that U is not bounded from below and D dimensional energy density diverges. ϕ goes to zero as ϕ' goes to ∞ (or as $y \rightarrow y_c$). Therefore, the potential V is not bounded from above. For $\alpha < 1$, from eq. (26) and (27), the leading term in the limit of $y \rightarrow y_c$ is the second term,

$$\begin{aligned} U_{\text{leading}} &= -c_3 \frac{1}{(y_c - y)^{2\alpha}} \\ &= -\bar{c}_3 \left(\frac{A}{\phi}\right)^m, \end{aligned} \tag{38}$$

where $m = \frac{2\alpha}{1-\alpha} > 2$ is positive and c_3 and \bar{c}_3 are also positive constants. As we approach y_c , the scalar field ϕ goes to zero and $U \rightarrow -\infty$. This is clear from the first line since we take the limit $y_c - y \rightarrow 0^+$. Thus the potential V goes to ∞ and is not bounded from above. By setting $x \sim \frac{1}{y_c - y}$, the " D dimensional energy density" formula is the same as for $\alpha > 1$ and diverges for $\alpha < 1$. Thus we confirm that for $\alpha < 1$ our criterion is not satisfied and also the potential is not bounded from above.

IV. DISCUSSION

The above observations can be summarized in the following ways. The criterion given here is fundamental since it is the necessary condition to have a consistent physical theory which allows us: to have a semi-classical expansion around the classical solution and to have a theory satisfying the consistency condition without invoking the infinite tension brane. The first property applies generally for the geometry without involving the brane (without cutting the AdS).

² AdS_5 supergravity has been studied in [25], and the region agrees with it for $D = 4$. ξ in [25] corresponds to $1/A$ but $\sqrt{2}$ appears due to the use of different units for 5 dimensional Planck scale.

All the examples studied here shows that two conditions, the finiteness of D dim. energy density and the potential bounded from above in the solution, are the equivalent. If we consider scalar potential which is bounded from above, D dimensional energy density of the solution remains finite. Also if we restrict our interests only on the solution whose D dimensional energy density is finite, the potential is bounded from above. It is puzzling that the two conditions which apparently look entirely independent are equivalent in all cases studied here. We can give partial answer to this puzzle. The anti-frictional force generated by either the potential energy or the initial velocity ϕ' destabilizes the scalar field, and there are two options:

1. $\phi \rightarrow \pm\infty$ and $V(\phi) \rightarrow \infty$
2. $\phi \rightarrow \pm\infty$ and $V(\phi) \rightarrow -\infty$.

In case 1, the Hubble parameter is given by the difference of ϕ'^2 and V , $h \sim \sqrt{\frac{\phi'^2}{2} - V}$, where both go to infinity but the difference can be less than that. In case 2, the Hubble parameter is given by the addition of ϕ'^2 and $|V|$, $h \sim \sqrt{\frac{\phi'^2}{2} + |V|}$, which is larger than $|V|$ itself. This gives rapid suppression of the warp factor and the effective energy density can remain finite even though $|V| \rightarrow \infty$. For the potential bounded from above, the ϕ'^2 and $|V|$ give additional contribution to warp factor enabling us to suppress the effective on-shell Lagrangian density. It is not easy to expect similar things in case 1 to make the energy density finite since the scalar field runs to infinity too fast and there is no chance to have a finite effective on-shell Lagrangian density by the suppression of the warp factor. Though no rigorous proof is available at this moment which can guarantee the equivalence between two conditions (finite energy density condition and the bounded above potential condition), all the examples considered in this paper show that two conditions are equivalent. In this sense, the approach used in this paper gives one explanation of why Gubser's conjecture is working.

We leave the rigorous proof confirming the relations among the various criteria given in [21,25–27] and the finiteness of D dimensional energy density, and the issue related to timelike naked singularity allowed in our criterion as future works.

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TABLE

Analogy of inflation models (de Sitter) and models in Anti-de Sitter space
in D dimensional spacetime (t, \vec{x}, y)

	Inflation/Deflation	AdS decreasing/increasing
Evolving parameter	t	y
Potential	$V(\phi(t))$	$-V(\phi(y))$
Metric	$-dt^2 + e^{2\int dt H_t}(d\vec{x}^2 + dy^2)$	$e^{2\int dy H_y}(-dt^2 + d\vec{x}^2) + dy^2$
Hubble parameter	$H_t = \pm \sqrt{\frac{2(\frac{\dot{\phi}^2}{2} + V)}{(D-1)(D-2)}}$	$H_y = \mp \sqrt{\frac{2(\frac{\phi'^2}{2} - V)}{(D-1)(D-2)}}$
Equation	$\ddot{\phi} + (D-1)H\dot{\phi} + \frac{dV}{d\phi} = 0$	$\phi'' + (D-1)H\phi' - \frac{dV}{d\phi} = 0$
Attractor condition	$V_{\min} > 0, H_t > 0$	$V_{\max} < 0, H_y > 0$
Einstein action	Exponential hierarchy	Exponential hierarchy
Brans-Dicke action	Power law hierarchy	Power law hierarchy

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